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DETACHMENT SIZE OF BUBBLES DURING QUASISTATIC GROWTH ON HEATER

Yu. A. Kirichenko, L. A. Slobozhanin, and N. S. Shcherbakova

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The critical bubble dimensions for detachment from a smooth surface and from the edge of a recess are calculated. The boundary of the quasistatic regime is estimated on the basis of the pressure.

One of the most important problems in the physics of boiling is that of determining the size of vapor bubbles as they detach from the heater. The detachment size and the nature of the detachment depend on the magnitude of the inertial forces in comparison with the forces due to gravity and surface tension. The inertial forces are governed primarily by the rate of growth of the bubble surface area and are proportional to β^4 [1]. Since β in turn is proportional to the m-th power of the Jacobi number ($0.5 \le m \le 1$) [2], and the Jacobi number is inversely proportional to ρ'', at high pressures the rapid increase in ρ'' causes β^4 to become extremely small. Under these conditions we can neglect the influence of inertial forces on the bubble detachment. The bubble growth is assumed to be quasistatic, and the detachment is assumed to begin at the instant the equilibrium free surface of the liquid becomes unstable.

Despite the importance of the problem of the detachment of bubbles during boiling, it has received little study, even in the case of quasistatic bubble growth. The average detachment radius $r_d = (3v_d/4\pi)^{1/3}$ is usually estimated from the equation [3]

$$r_d \cong 0.01 \sqrt{\frac{\sigma}{(\rho-\rho'')g}}\,\theta \quad (r_d \cong 0.01\, b^{-1/2}\theta;\ b = (\rho-\rho'')\,g/\sigma). \tag{1}$$

This equation was derived by Fritz [4] for a bubble sitting on a smooth horizontal plate; Fritz determined v_d as the maximum bubble volume (for a given value of θ) and used the tables of Bashforth and Adams [5] to construct the function $V_d(\theta)$ for $\theta \ge 59°$ ($V_d = v_d b^{3/2}$). We note that Eq. (1) becomes unacceptably inaccurate at $\theta > 125°$, according to the results of [4]. Furthermore, the applicability of (1) for $\theta < 59°$ has yet to be proved.

Nesis and Komarov [6] proposed an approximate analytic solution of this problem for small θ on the basis of a study of the behavior of the base radius X as a function of the bubble height h. They asserted that for $\theta < 70°$ the condition $dX/dh = -\infty$ becomes satisfied upon the appearance of an inflection point of the generatrix of the bubble surface at the base; this condition was adopted as the stability threshold. Analytic and numerical calculations in [10] showed that the inflection point does not coincide with the point at which we have $dX/dh = -\infty$. Furthermore, the satisfaction of the condition $dX/dh = -\infty$ cannot be judged a rigorous criterion for the loss of stability; as follows from a numerical solution of the variational problem, the instability occurs upon the appearance of an inflection point on the profile, but before the time at which the condition $dX/dh = -\infty$ becomes satisfied [10, 14].

For liquids with very small wetting angles (e.g., for cryogenic liquids, for which $\theta \approx 0°$), Eq. (1) predicts values of r_d which are far too low. It is reasonable to assume that the vapor bubble does not detach from the smooth wall of the heater but from the edge of a microscopic recess [1]. The most likely place for the nucleation of a bubble is at the bottom of a microscopic recess. As the bubble grows, the base of the bubble

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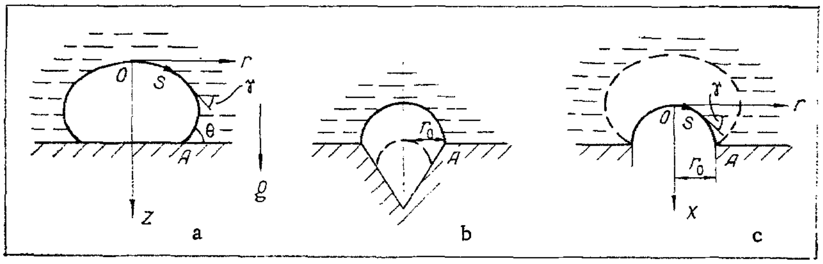

Fig. 1. Detachment models. a) Bubble on smooth plate; b) at microscopic recess; c) on plate with circular aperture.

(more precisely, the wetting line, i.e., the line of intersection of the free surface with the heater) moves along the lateral surface of the recess [7]. The shape of the microscopic recess (the most realistic model is a conical surface [7]) is important only for the nucleation and growth of the bubble within the recess. Using the methods and results of [8], we can easily reach the conclusion that the detachment of a bubble from a lateral surface of a cone with the very small base corresponding to real microscopic recesses is impossible. (According to estimates in [9], the size of microscopic recesses is on the order of 1-100 μ.) At a certain instant, accordingly, the bubble must settle on the edge of the microscopic recess (Fig. 1b). After this occurs (Fig. 1c), the bubble shape changes in different manners, depending on the ratio of θ and R_0 ($R_0 \equiv r_0 \times \sqrt{(\rho - \rho'')g/\sigma}$). The detachment can occur either from a smooth wall or from the edge of the recess [10].

Buevich and Butkov [11] took up the problem of finding the critical volume of a bubble which detaches from the edge of an aperture (the bubble volume in this case was taken to be the volume founded by the free surface and the horizontal plane of the aperture). Working by analogy with [4], they determined the detachment volume as the maximum volume for the equilibrium states of the bubble, for which the wetting line has a fixed radius r_0, equal to the aperture radius. They constructed the function $V_d(R_0)$ for $R_0 \leq 2.5$.

It should be noted here that the method used by Fritz leads to a correct solution of the problem he dealt with, since in this case only the axisymmetric perturbations leading to bubble detachment are important [8], and the instant at which these perturbations cause instability coincides with the instant at which the bubble reaches its maximum volume (for a given value of θ; this result was shown rigorously in [12]). At the same time, the method used by Buevich and Butkov [11] is valid only for $R_0 \leq 3.24$, at which axisymmetric perturbations are dangerous, and this method leads to incorrect results if $3.24 < R_0 \leq 3.83$, in which case the instability due to nonaxisymmetric perturbations occurs earlier [10] (if $R_0 > 3.83$, the bubble is always unstable).

Furthermore, the approximation of the results by the function $V_d = 0.65R_0$ used in [11] is clearly erroneous.

Slobozhanin and Tyuptsov [10] numerically determined the bubble shape during slow growth up to the instant at which detachment begins, and they found the detachment volume (we emphasize that "detachment volume" here, as in [4, 11], is understood to be – not the volume of the bubble after it has detached – but the volume of the bubble at the beginning of detachment, i.e., at the instant at which the instability occurs; the volume after detachment probably cannot be found exactly without solving the dynamic problem). They dealt with the cases of a smooth horizontal plate (Fig. 1a) and a plate having a circular aperture (Fig. 1c). For the second case they analyzed the conditions under which the detachment occurs from the smooth surface and from the edge of the aperture. In solving the detachment problem they used the methods of [8, 13, 14]. They constructed the functions $V_d(R_0)$ for detachment from the edge of the aperture and $V_d(\theta)$ for detachment from the smooth surface.

In the present paper we calculate the interval of small wetting angles and aperture radii given roughly in [10], and we find simple equations to approximate the numerical results found in the present paper and, partially, in [10]. Then it becomes possible to determine the detachment sizes of bubbles for various liquids (including cryogenic liquids) by adopting as a model for a flat heater surface a plate with a circular aperture. Furthermore, we use the pressure to estimate the boundary between the quasistatic and dynamic regimes for bubble growth and detachment.

We introduce a cylindrical coordinate system r, φ, z with origin at the crest of the bubble and with z axis pointing downward, along the direction of the gravitational force (Fig. 1). By virtue of the axial symmetry of the problem, in order to determine the bubble shape at some instant it is sufficient to determine the shape of the generatrix – the line of intersection of the bubble surface with the φ = const half-plane. We call this

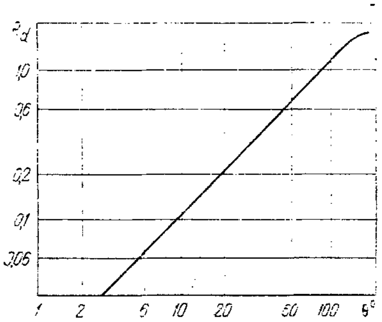

Fig. 2

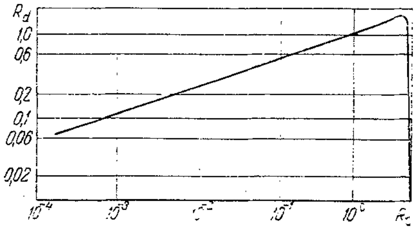

Fig. 3

Fig. 2. Dimensionless bubble detachment radius R_d as a function of the wetting angle $\theta°$.

Fig. 3. Dimensionless detachment radius R_d as a function of the dimensionless radius of the recess, R_0.

generatrix the "equilibrium line" (Fig. 1). Transforming to dimensionless variables, we find that the equilibrium line is described in the parametric form R(S), Z(S) by the system of equations [8]

$$R'' = -Z'(-Z + C - Z'/R),\ Z'' = R'(-Z + C - Z'/R),\ (' = d/dS) \tag{2}$$

with the inertial conditions

$$R(0) = Z(0) = Z'(0) = 0,\ R'(0) = 1. \tag{3}$$

The solutions of problem (2), (3) form a single-parameter family of integral curves (the parameter is the quantity C). The equilibrium line is governed not only by the value of the parameter C, which governs the shape of the integral curve, but also by the position of the final point $S = S_1$ and the point of contact on this curve, A (Fig. 1). These two quantities, which are not known at the outset, can be found from the condition for a given bubble volume (at a given instant) and from the condition for a given wetting angle,

$$\gamma(S_1) = \pi - \theta, \tag{4}$$

if the bubble is sitting on a smooth surface, or from the condition

$$R(S) = R_0, \tag{5}$$

if the bubble is in contact with the edge of an aperture.

The bubble can be in contact with the edge of the aperture only if [13]

$$\gamma(S_1) \leqslant \pi - \theta. \tag{6}$$

To study the stability of the bubble we begin with the principle of a minimum potential energy of the system. The problem reduces to one of determining the sign of the smallest eigenvalue of a linear boundary-value problem for the normal component $N(S, \omega)$ of the perturbations of the free surface [8, 14]. The coefficients in this problem depend on the shape of the bubble surface (and, of course, on the physical properties determining the equilibrium).

The boundary conditions for this problem take different forms, depending on whether the bubble is in contact with a smooth wall or with the edge of an aperture: in the first case, the boundary condition expresses the conservation of the wetting angle during tolerable perturbations; in the second case, it expresses the conservation of the contact line at the edge of the aperture. Accordingly, the detachment volumes will be different in these two cases.

Using the specified procedure, we numerically determined the detachment sizes for bubbles in contact with the smooth surface of a plate for small wetting angles (the extreme value was $\theta = 3°$). On the basis of these results and the results of [10] we were then able to construct the function $R_d(\theta)$ shown in Fig. 2. Comparison of this function with Eq. (1) shows that over the interval $3° \le \theta \le 125°$ Eq. (1) approximates the exact results within a relative error no larger than 5%. In the same interval of wetting angles, this function is described more accurately (within 1.5%) by

$$R_d = 0.01060\theta \left(r_d = 0.01060\theta \sqrt{\frac{\sigma}{g(\rho - \rho'')}} \right). \tag{7}$$

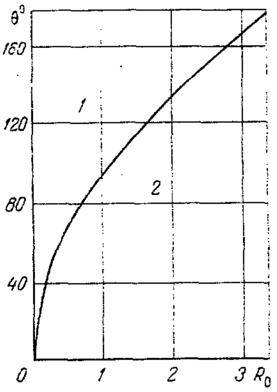

Fig. 4

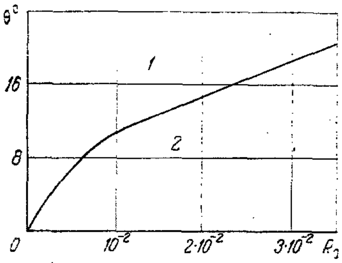

Fig. 5

Fig. 4. Boundary between the two detachment regimes. 1) Detachment from smooth surface; 2) detachment from edge of recess.

Fig. 5. The part of the boundary in Fig. 4 for small values of R_0 and θ.

Furthermore, for the case of detachment from the edge of a recess we numerically found the detachment dimensions for small values of R_0 (the smallest value of R_0 for which calculations were carried out was $1.8\cdot 10^{-4}$). Figure 3 shows the function $R_d(R_0)$, in the plotting of which we also used the results of [10].

Over the interval $1.8\cdot 10^{-4} \leq R_0 \leq 0.5$ this function can be approximated by

$$R_d = 1.105\sqrt[3]{R_0}\left(r_d = 1.105\sqrt[3]{\frac{r_0\sigma}{g(\rho-\rho'')}}\right). \tag{8}$$

The error of this approximation is less than 2.5%.

How does a bubble detach from a heater with a recess – from the smooth surface or from the edge of the recess? Figure 4 gives us an answer to this question. In this figure, the boundary between the region of physical parameters in which the detachment occurs from the edge of the recess, 2, and the region in which detachment occurs from the smooth surface, 1, is shown on the (R_0, θ) plane. Figure 5 shows the part of the boundary for small values of R_0 and θ, which is a particularly interesting situation.

We see, in particular, from these figures that for cryogenic liquids, with extremely small wetting angles, Eqs. (1) and (7) are inapplicable, since in these cases we are dealing with detachment from microscopic recesses. (This assertion is confirmed by the experimental results of [15], which showed that we have $r_d \sim g^{-1/3}$ for liquid oxygen, in agreement with Eq. (8).)

These results can be used to analyze boiling if we know the average radius of the microscopic recess, r_0, and if we have determined the boundary of the quasistatic regime for the detachment of vapor bubbles during bubble boiling. For definiteness we assume that the radii of the active vapor-formation centers, r_0, are equal to their most probable value [16]:

$$r_0 = \frac{4\sigma T_s}{L\rho''\Delta T}. \tag{9}$$

The boundary between the quasistatic and dynamic detachment regimes can be determined by comparing the surface-tension force, which can be written (within accuracy sufficient for our purposes) as $F_\sigma = 2\pi r_0\sigma$ for the case in which the bubble detaches from the edge of a microscopic recess, with the inertial force of the reaction of the liquid, $F_R = (\pi/3)\beta^4\rho$ [1]. From the condition $F_\sigma = F_R$ we find the boundary value of the growth modulus β_* and then the boundary value of the temperature difference ΔT_* (the dependence of β on ΔT for the cryogenic liquids is determined from the equation given by Labuntsov et al. [2], while that for water and ethanol is determined from the Cole–Shulmann equation [17]). Comparing the difference ΔT_* with the pressure dependence of the temperature difference for the boiling of certain liquids [18, 19], we find the boundary between the quasistatic and dynamic detachment regions in terms of the pressure P_* (or in terms of the reduced pressure $\pi_* = P/P_c$). The calculated results are shown in Table 1. Here the values of $\pi_{*\,\min}$ were determined for heat flux densities q corresponding to the beginning of boiling, while the values of π_* are determined for $q \approx 0.5q_{cr}$.

TABLE 1. Boundary Values of the Reduced Pressure for Various Substances

Substance	He	H_2	N_2	O_2	C_2H_5OH	H_2O	Average
π_{*} min	0,03	0,05	0,03	0,02	0,02	0,015	0,03
π_{*}	0,1	0,1	0,06	0,04	0,03	0,03	0,06

The boundary found between the quasistatic and dynamic detachment regimes for vapor bubbles, and thus the boundary between "high" and "low" pressures, must obviously be refined.

The values of r_0 which we used in calculating π_* are apparently near the minimum possible values for active vapor-formation centers [9]. For example, for water at atmospheric pressure and with $\Delta T \approx 10$ deg, estimate (9) gives $r_0 \approx 7\ \mu$. The values of r_0 actually observed under these conditions are on the order of 30 μ [20, 21]. On the basis of the results shown in Table 1, however, we can assume that the results obtained for quasistatic detachment are applicable at $\pi > 0.1$. If, on the other hand, we have $\pi < 0.01$, then the vapor bubbles detach in the dynamic regime. In the latter case, and in the interval $0.01 < \pi < 0.1$, the detachment dimensions can be estimated by the procedure of [1].

NOTATION

β, growth modulus; ρ, ρ'', densities of liquid and vapor; θ, wetting angle of liquid; σ, surface-tension coefficient; g, acceleration due to gravity; r_d, v_d, detachment radius and detachment volume of bubble; r_0, base radius of microscopic recess (or radius of the aperture in the plate); $b^{-1/2}$, scale linear dimension; R_d, V_d, dimensionless detachment radius and detachment volume; R_0, dimensionless radius of microscopic recess; T_s, saturation temperature; ΔT, temperature difference; L, latent heat of vaporization; P, P_c, π, vapor pressure, critical pressure, and reduced pressure; β_*, ΔT_*, P_*, π_*, boundary values of the growth modulus, the temperature difference, the pressure, and the reduced pressure; q_{cr}, first critical heat flux density.

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